

# Gauge Five Brane Dynamics And Small Instanton Transitions In Heterotic Models

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We present the first examples of cosmological solutions to four-dimensional heterotic models which include an evolving bundle modulus. The particular bundle modulus we consider corresponds to the width of a gauge five brane. As such our solutions can be used to describe the evolution in one of these models after a small instanton transition. We find that certain properties are generic to these solutions, regardless of initial conditions. This enables us to make some definite statements about the dynamics subsequent to a small instanton transition despite the fact that we cannot microscopically describe the process itself. We also show that an effective description of the small instanton transition by a continuous matching of fields and their first derivatives is precluded by the form of the respective low-energy theories before and after the transition.

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## I. INTRODUCTION

A small instanton transition takes place when an M5 brane, moving in the orbifold direction of heterotic M-theory [1–6], impinges upon one of the fixed planes [7,8]. It is thought that when such a collision occurs the M5 brane could disappear and be replaced with a gauge five (G5) brane living upon the relevant fixed plane.

It is still not known whether small instanton transitions are dynamically allowed. By this we mean does the G5 brane, which initially appears with a width at the fundamental length scale, actually spread out with time to become an object describable by supergravity. Certainly we know that the process is consistent with conservation of various charges [9]. We also know that the extra light states which appear as the M5 brane approaches the fixed point are the same as those which arise due to a G5 brane's core shrinking to zero size [7,8]. However, to determine whether dynamically this transition actually completes itself would require an understanding of the process at times when either the instanton is small when compared to the fundamental length scale or, if we approach the problem from the other side, when the M5 brane is within a fundamental length scale distance of the orbifold fixed plane. This is an extremely difficult task due to the presence of a tensionless non-critical string which appears during the transition.

To make clear what we will achieve in this paper let us split a small instanton transition as described above into three distinct regimes.

- Firstly we have the regime where the M5 brane is moving in the bulk toward the orbifold fixed plane but has not yet come sufficiently close for the extra states associated with membranes attached to the M5 and its orbifold mirror or the fixed point to become light. The dynamics associated with this regime have been described in detail in [10,11]. However, even though this regime has been studied before we will find that, in order to describe the dynamics for the same compactification throughout, we need to derive some new solutions. This is because, as we will discuss in section II, the existing four dimensional moving five brane solutions [10] are not compatible with being connected via a small instanton transition to solutions of the known G5 brane effective actions [12]. We therefore derive some moving M5 brane solutions which are compatible with the solutions we shall present for regime three to illustrate that the qualitative behaviour as described in previously examined cases still holds here.
- Secondly we have the regime of the small instanton transition itself where the tensionless non-critical string has to be taken into account. We shall have nothing to say directly about this regime in this paper.
- Lastly we have the regime where the G5 brane size has dilated enough for the normal low energy supergravity description to be valid (i.e. we are assuming that the transition is indeed dynamically allowed). The description of the dynamics associated with this regime is the main result of this paper. The evolution of the G5 brane after such a transition has not been investigated before. This is because the four dimensional theory describing the necessary degrees of freedom has only recently been derived [12].

Even though we will not be able to say anything new here about the microscopic dynamics in the second of these regimes we will find that we can still say something concrete about what happens after a small instanton transition, under the assumption that it is indeed not dynamically excluded. The solutions which describe regime three have certain properties which are generic whatever choices of initial conditions are made for the moduli fields after collision. This allows us to make some qualitative statements about the dynamics after a small instanton transition despite the fact that we do not know how the solutions of regime one and three are matched at the point of collision and so we do not know what initial conditions to give to our solutions in regime three.

We will also show that the simplest possible effective description of the small instanton transition, corresponding to a continuous matching of the various fields and their first derivatives across regime two, is precluded. This is due to an incompatibility of this procedure with the Hamiltonian constraints on our solutions before and after the transition.

We would also like to stress that G5 branes can exist on the orbifold fixed planes regardless of whether or not there has been a small instanton transition in the past. As such the solutions we provide for regime three are interesting in their own right. In particular they constitute the first examples of cosmological solutions to heterotic models which include a gauge bundle modulus.

It may not be clear how the freely moving solutions we present here are compatible with recent work on moduli stabilisation in heterotic models [13]. We assume that all of the potentials we are neglecting are compatible with the standard four dimensional heterotic effective theory obtained by reduction on the vacua of [5,14]. Furthermore we require that the energy scales associated with the stabilisation potentials, for the values of moduli as given by our solutions, are several orders of magnitude smaller than the energy scale at which the four dimensional effective theory breaks down. We can then take the kinetic energies in our solutions to be large when compared to the potentials we are ignoring - thus justifying our approximation. One way in which this approximation can always be consistently achieved is to take the large radius limit.

The outline of this paper is as follows. In section II we give the effective actions which are necessary to describe the four dimensional physics before and after the small instanton transition. In section III we present the cosmological solutions associated with these actions. Section IV contains a discussion of what conclusions we can draw about the dynamics after a small instanton transition, an explanation of how we can rule out the simplest possible effective description of the small instanton transition and a brief outline of some possible generalisations of our solutions.

## II. THE FOUR-DIMENSIONAL EFFECTIVE ACTIONS

M5 branes in the vacuum of heterotic M-theory have to be oriented in a certain way in order to preserve  $N = 1$  supersymmetry in the four dimensional theory [3]. Four of the five brane's six dimensions have to span the external Minkowski space. The remaining two world volume directions must then wrap a holomorphic curve within the Calabi-Yau. This leaves five dimensions in total which are transverse to the five brane. One of these is the orbifold direction and the remaining four lie within the Calabi-Yau.

The configuration of the G5 brane which appears after the collision is constrained by the requirement that a cohomology condition, required for consistency of the vacuum solution, should still be satisfied in the M5 brane's absence. This condition essentially encodes the need for the net magnetic charge on various compact manifolds to be zero. It is easy to see, using the expressions given in [3] for example, that these restrictions mean that the G5 brane after the collision must be wrapped in the same way as the M5 brane beforehand.

In the regime before the collision with the orbifold fixed plane the four dimensional action is known for any compactification manifold of  $SU(3)$  holonomy and for the case of the five brane wrapping an arbitrary holomorphic curve within that manifold [15–18].

In the regime after the collision our knowledge is more restricted. The only known case (which also represents the only known examples of kinetic terms for bundle moduli) was presented in [12]. For this result to be valid the compactification manifold is required to have one crucial property. Near the two cycle which the G5 brane

wraps the manifold must be decomposable as a direct product of the two cycle and a four dimensional complex transverse space. A class of examples of such manifolds (which in addition should have  $SU(3)$  holonomy of course) were provided in [12]. These examples were based on Calabi-Yau threefolds constructed as the resolution of certain six dimensional orbifolds.

Given this restricted knowledge we should choose our compactification manifold and holomorphic curve in regime one such that we can write down the compatible theory, based upon the same compact manifold and holomorphic curve in regime three. Unfortunately the only four dimensional moving M5 brane solutions in the literature [10] are for the case  $h_{1,1} = 1$  which is not compatible with this requirement. We will therefore derive a set of solutions for regime one, as well as regime three which, however, remains the case of primary interest in this paper. We shall see that all of the qualitative features of the moving M5 brane solutions in the  $h_{1,1} = 1$  case, [10], are reproduced for the solutions we shall present.

To be concrete we choose as our Calabi-Yau threefold a resolved  $Z_8$  - I Coxeter orbifold with an  $SO(5) \times SO(9)$  lattice [19]. The homology class of the holomorphic curve wrapped by the five branes is then taken to be that associated with a wrapping of the subspace which undergoes identifications by the  $SO(5)$  part of the lattice.

The appropriate four dimensional actions are then as follows. For regime three when the G5 brane is present we have [12],

$$S = \frac{1}{2\kappa_P^2} \int \sqrt{-g} \left[ -R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \beta_3 \partial^\mu \beta_3 + \partial_\mu \beta \partial^\mu \beta + 8q_{G5} e^{-\beta} \partial_\mu \hat{\rho} \partial^\mu \hat{\rho} \right]. \quad (1)$$

For regime one when the M5 brane is present we have [15–18],

$$S = \frac{1}{2\kappa_P^2} \int \sqrt{-g} \left[ -R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \beta_3 \partial^\mu \beta_3 + \partial_\mu \beta \partial^\mu \beta + q_{M5} e^{\beta_3 - \phi} \partial_\mu z \partial^\mu z \right]. \quad (2)$$

In these actions  $\phi$ ,  $\beta$  and  $\beta_3$  are moduli describing the compactification space (we have of course performed many consistent truncations from the full set of four dimensional fields). The modulus  $\beta$  determines the size of the four dimensional space transverse to our five branes in the Calabi-Yau from the perspective of the ten dimensional effective theory. The other Calabi-Yau modulus,  $\beta_3$ , determines the size of the two cycle the five branes wrap. The four dimensional dilaton is denoted by  $\phi$ . The kinetic terms for these moduli are the same in both cases because while the two different actions represent different physics they are based upon compactifications on the same internal manifold.

We also have the moduli fields that describe the relevant degrees of freedom of our five branes. The coordinate position of the M5 brane within the orbifold direction is denoted by  $z$ . The orbifold fixed planes are at  $z = 0$  and  $z = 1$  in our coordinates. The width of the G5 brane in the first of the two actions is denoted by  $\hat{\rho}$ . In fact this is the coordinate size of the G5 brane where our conventions mean that the coordinate size of the transverse space in the Calabi-Yau is a constant,  $v_{\text{trans}}^{\frac{1}{4}}$ . As such  $\hat{\rho}$  can vary from 0 up to  $v_{\text{trans}}^{\frac{1}{4}}$  as it expands in the transverse space. When plotting specific examples of our solutions we shall take  $v_{\text{trans}}^{\frac{1}{4}} = 1$ .

The factor of  $e^{\beta_3}$  in the prefactor to the  $z$  kinetic term in (2) is a somewhat non-trivial consequence of the general expressions presented in [16,18] and the nature of the holomorphic curve which our M5 brane is wrapping. This latter, as we have stated above, is determined by the need to match the configurations in regime one and three in a manner which is consistent with the cohomology condition and the action (1).

Now that we have written down the appropriate effective theories in the two regimes of interest we can proceed to find the cosmological solutions that form the core of the results of this paper.

### III. COSMOLOGICAL SOLUTIONS INCLUDING GAUGE-FIVE BRANE OR M-FIVE BRANE MODULI

We wish to investigate the cosmological dynamics which follow from the actions (1) and (2) presented in the previous section. In particular we are interested in the effects that a moving M5 brane or an expanding or contracting G5 brane can have on the evolution of the other four dimensional fields. The procedure for obtaining cosmological solutions from such actions is well known [20] and so we shall not go into detail regarding this.

We make the ansatze,

$$ds^2 = -e^{2\nu}d\tau^2 + e^{2\alpha}d\mathbf{x}^2 \quad (3)$$

where  $\alpha, \beta, \beta_3, \phi, \hat{\rho}$  and  $z$  are all functions of  $\tau$  only. The three-dimensional spatial submanifold is taken to be Ricci flat and we have either a  $\hat{\rho}$  or a  $z$  modulus depending on which case we are considering.

The equation of motion for the size (position) of the G5 (M5) brane can be trivially integrated once.

$$\dot{\hat{\rho}} = \tilde{s} e^{\beta+\nu-3\alpha} \quad (4)$$

$$\dot{z} = \tilde{d} e^{\phi-\beta_3+\nu-3\alpha} \quad (5)$$

Here  $\tilde{s}$  and  $\tilde{d}$  are arbitrary integration constants. These expressions can then be used in the remaining equations of motion to obtain a closed system in terms of  $\alpha, \beta, \beta_3$  and  $\phi$ . The resulting equations can be written in the following compact form.

$$\frac{d}{d\tau}(EG\boldsymbol{\alpha}') + E^{-1}\frac{\partial U}{\partial \boldsymbol{\alpha}} = 0 \quad (6)$$

$$\frac{1}{2}E\boldsymbol{\alpha}'^T G\boldsymbol{\alpha}' + E^{-1}U = 0, \quad (7)$$

The relevant four dimensional fields are arranged in a vector  $\boldsymbol{\alpha} = (\alpha, \beta, \beta_3, \phi)$ . The “Einbein”  $E$  is defined as  $E = e^{-\nu+\mathbf{d}\cdot\boldsymbol{\alpha}}$ , where we have defined the vector  $\mathbf{d} = (3, 0, 0, 0)$ . The potential  $U$  arises as a direct result of the presence of the dynamical G5 (M5) brane. It is determined by a constant  $u$  and a vector  $\mathbf{q}$  which characterises the particular case under study.

$$U = \frac{1}{2}u^2 e^{\mathbf{q}\cdot\boldsymbol{\alpha}}. \quad (8)$$

In the G5 brane case  $u$  and  $\mathbf{q}$  take the following values.

$$u^2 = 4q_{G5}\tilde{s}^2 \quad (9)$$

$$\mathbf{q} = (0, 1, 0, 0) \quad (10)$$

In the M5 brane case they take a different form. This is due to the kinetic term for the brane’s position modulus coupling differently to the other four dimensional fields.

$$u^2 = \frac{1}{4}q_{M5}\tilde{d}^2 \quad (11)$$

$$\mathbf{q} = (0, 0, -1, 1) \quad (12)$$

In both cases the metric  $G$ , which is simply determined by the coefficients of the kinetic terms of the components of  $\boldsymbol{\alpha}$  as given in the actions (1), (2), takes the same form.

$$G = \text{diag}(-3, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}) \quad (13)$$

By a judicious choice of gauge we can find the general solution to this system of equations in both cases. We shall now present these, starting with the G5 brane case.

### A. Solutions including gauge five-brane moduli

We find the following solutions describing the evolution of the gauge five brane's size modulus and the other four dimensional fields. Our solutions are presented in comoving gauge for ease of physical interpretation.

$$\alpha = \frac{1}{3} \ln \left| \frac{t - t_0}{T} \right| + \alpha_0 \quad (14)$$

$$\beta = p_{\beta,i} \ln \left| \frac{t - t_0}{T} \right| + (p_{\beta,f} - p_{\beta,i}) \ln \left( \left| \frac{t - t_0}{T} \right|^{-\delta} + 1 \right)^{-\frac{1}{\delta}} + \beta_0 \quad (15)$$

$$\beta_3 = p_{\beta_3,i} \ln \left| \frac{t - t_0}{T} \right| + \beta_{30} \quad (16)$$

$$\phi = p_{\phi,i} \ln \left| \frac{t - t_0}{T} \right| + \phi_0 \quad (17)$$

$$\hat{\rho} = \hat{s} \left( 1 + \left| \frac{T}{t - t_0} \right|^\delta \right)^{-1} + \hat{\rho}_0 \quad (18)$$

It is reassuring to note that the expansion power for the four dimensional scale factor is  $1/3$ , as expected for kinetic energy driven expansion in the Einstein frame. The constants  $p_{\beta,n}$ ,  $p_{\beta_3,n}$  and  $p_{\phi,n}$  are subject to the constraint,

$$2p_{\beta,n}^2 + p_{\beta_3,n}^2 + p_{\phi,n}^2 = \frac{4}{3}, \quad (19)$$

for  $n = i, f$ . In this expression  $p_{\beta_3,f} = p_{\beta_3,i}$  and  $p_{\phi,f} = p_{\phi,i}$ . We shall see the physical meaning of (19) shortly. The constants,  $\beta_0$  and  $\hat{s}$  are subject to a constraint.

$$\beta_0 = \ln(2q_{G5}\hat{s}^2) \quad (20)$$

The constant  $\delta$  takes the following form.

$$\delta = -p_{\beta,i} \quad (21)$$

Finally we have the arbitrary integration constants  $\hat{\rho}_0$ ,  $\alpha_0$ ,  $\beta_{30}$ ,  $\phi_0$ ,  $t_0$  and  $T$ .

The above class represents the first examples of cosmological solutions to heterotic models which include a gauge bundle modulus. As such they are of interest in their own right as well as in the more specific context we are considering in this paper. They detail some of the kind of effects we can expect a dynamical gauge bundle to have on the cosmological evolution of the theory.

We shall now spend some time describing in detail the physics which these solutions represent. The broad features presented may be familiar to some readers as formal analogies can be drawn between the system of current interest and systems involving potentials due to form fields [20] or moving M5 branes [10].

We first consider the limitations placed on the range of  $t$  in the above solutions by the requirement that the logarithms remain well defined. We find two possible allowed ranges.

$$t \in \begin{cases} [-\infty, t_0] , & (-) \text{ branch} \\ [t_0, +\infty] , & (+) \text{ branch} \end{cases} \quad (22)$$

Thus we find a separation of our cosmological solutions into so called positive and negative time branches. The negative time branch starts with an asymptotically flat universe and evolves into a curvature singularity at

$t = t_0$  whereas the positive time branch starts at a curvature singularity at time  $t_0$  and evolves to less and less highly curved configurations with time. In this paper we shall focus, in our discussions, on positive time branch solutions. The reader who wishes to consider the negative time branch case need only take the time reverse of our analysis.

There is a redundancy in the constants labelling our solutions. This is due to a symmetry under the following transformations.

$$\delta \rightarrow -\delta \quad (23)$$

$$\mathbf{p}_i \rightarrow \mathbf{p}_f \quad (24)$$

$$\mathbf{p}_f \rightarrow \mathbf{p}_i \quad (25)$$

$$\hat{s} \rightarrow -\hat{s} \quad (26)$$

$$\hat{\rho}_0 \rightarrow \hat{\rho}_0 + \hat{s} \quad (27)$$

The physical situation described is unchanged under this change of constants. To avoid over counting solutions we shall therefore employ the following convention.

$$\delta < 0 \quad (+)\text{branch} \quad (28)$$

$$\delta > 0 \quad (-)\text{branch} \quad (29)$$

Now let us consider how the various fields evolve in time. It can be seen from equation (18) that at either end of the  $t$  range, as  $t \rightarrow t_0$  or as  $t \rightarrow \infty$ ,  $\hat{\rho}$  approaches a constant. Now  $\hat{\rho} = e^{-\frac{\beta}{2}}\rho$  where  $\rho$  is the physical width of the gauge five brane. The other factor in  $\hat{\rho}$ ,  $e^{\frac{\beta}{2}}$ , measures physical lengths in the transverse space to the G5 brane in units of a fixed coordinate size for that submanifold. Thus  $\hat{\rho}$  being a constant has the physical meaning that the gauge five brane size is scaling with the size of the transverse portion of the compactification manifold. Any change in the soliton's width is due to an overall change in the size of the transverse space and not a change in size of the object relative to its surroundings.

While the G5 brane is of 'constant size' in this manner its kinetic term drops out of the effective four dimensional theory (1). Since this kinetic term was the only indication from our four dimensional perspective that the gauge five brane was present (we are not interested here in four dimensional gauge groups etc.) this means that asymptotically, when the size is constant, the solutions should approach those that have been obtained in the study of four dimensional heterotic M-theory in the absence of gauge five brane moduli [21].

An examination of the solutions (14) - (18) shows that this is indeed the case. We find that when  $t \rightarrow t_0$  we have a so called rolling radius solution with expansion powers for the geometrical moduli given by  $\mathbf{p}_i$ , whereas when  $t \rightarrow \infty$  we have another, in general different, such solution characterised by expansion powers  $\mathbf{p}_f$ . As is usual these expansion powers are subject to a constraint (19).

At times intermediate between the two asymptotic regions described above it is clear that  $\hat{\rho}$ , and so the size of the five brane relative to the compactification space, changes with time. In fact the size of the soliton monotonically changes from its initial value,  $\hat{s} + \hat{\rho}_0$ , to its final value,  $\hat{\rho}_0$ , at some time determined by  $T$ . As it does so it affects, via the non-trivial coupling of its kinetic term to  $\beta$ , the other moduli, which can be seen to no longer take the form of a simple rolling radius solution at these times. In fact, equation (15) tells us that the five brane motion maps the initial rolling radius solution into a final one determined by the mapping given below.

$$\begin{pmatrix} p_{\beta,f} \\ p_{\beta_3,f} \\ p_{\phi,f} \end{pmatrix} = P \begin{pmatrix} p_{\beta,i} \\ p_{\beta_3,i} \\ p_{\phi,i} \end{pmatrix}, \quad P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (30)$$

In particular the final set of expansion powers is completely determined by the initial one. The mapping given above is its own inverse. This is simply a consequence of the time reversal symmetry which exists in our four dimensional effective action (1).

It should be noted that, if we are in a positive time branch for example, not all of the possible rolling radius solutions are available to us as initial configurations at  $t \rightarrow t_0$ . The rolling radii configurations which are allowed as initial states for solutions with changing G5 brane width are those with  $p_{\beta,i} > 0$  (in the positive time branch case). In other words the transverse space to the G5 brane within the Calabi-Yau is initially expanding and collapses at late times.

There are a few special solutions in the class presented here for which  $\hat{\rho}$  is constant throughout the evolution. These correspond to solutions where the transverse space is constant in size,  $p_{\beta,i} = 0$ . Such configurations are indistinguishable in four dimensions from situations in which the G5 brane is not present.

We should at this point, make a few comments about when our four dimensional theory, and so the solutions we have derived in this section, is valid. We of course have all of the usual restrictions on the validity of the four dimensional effective description of heterotic M-theory. We also have an additional restriction in that the theory is also not valid when the G5 brane's width becomes comparable to the size of the transverse space. This is due to an approximation that was made in obtaining the four dimensional theory [12].

Let us consider in detail the requirement that the extra dimensions should be larger than the fundamental scale. Consider the constraint on the integration constants (20), the value of  $\delta$  (21) and the expansion power  $p_{\beta,f}$  in terms of the initial expansion powers (30). If we use all of this information in equation (15) for  $\beta$  we obtain the following.

$$e^\beta = \frac{2\alpha'(2\pi)^2\hat{s}^2}{v_{trans}} \left[ \left| \frac{t-t_0}{T} \right|^{p_{\beta i}} \left( \left| \frac{t-t_0}{T} \right|^{p_{\beta i}} + 1 \right)^{-2} \right] \quad (31)$$

The function in the square brackets in equation (31) is always less than 1.

For the length scale associated with the transverse space to be much greater than the fundamental scale we require,

$$e^\beta \sqrt{v_{trans}} \gg \alpha' \quad (32)$$

We see from equation (31) that the only way to achieve this is to have  $\hat{s} \gg v_{trans}^{\frac{1}{4}}$ . In other words we need the total change in size of the G5 brane during the evolution to be orders of magnitude greater than the size of the compact space in which it lives. This, of course, means that the full solutions with all of the behaviour described above are not valid in any one particular case. We can only follow the evolution for a certain period while the size of the G5 brane is bigger than the fundamental scale and smaller than the size of the compactification space. It should be stressed however that by a judicious choice of the constant  $\hat{\rho}_0$  any region of the solutions illustrated above can fall within this describable range. We just do not have valid solutions where we can follow the dynamics through from one asymptotic regime to the other.

It should be noted that because the transverse space always collapses asymptotically in these solutions our description always breaks down at late times.

Examples of the dynamics of the bundle moduli that we have obtained here are plotted, including the regimes where our four dimensional description is valid, in figures 1 and 2.



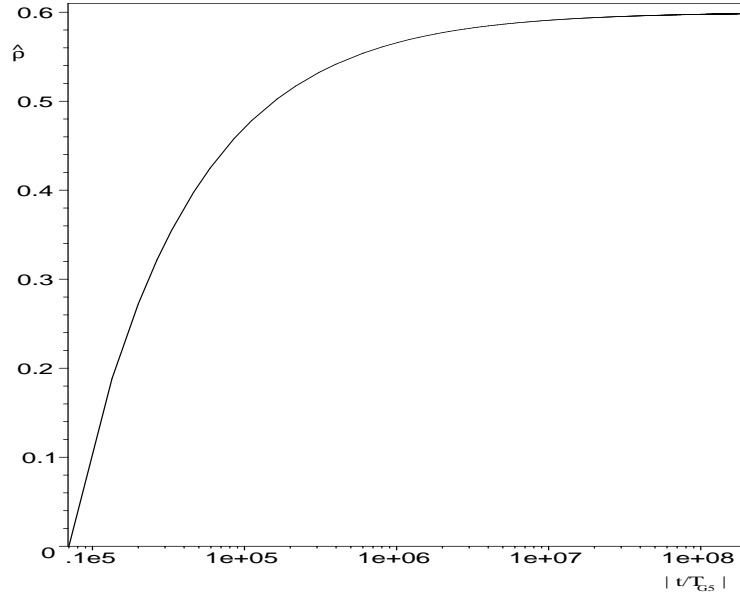


FIG. 1. Plot of the coordinate size of a gauge five brane including the regime where our four dimensional effective theory is valid. We have chosen the set up  $T_{G5} = 1$ ,  $\hat{s} = -100$  and  $\hat{\rho}_0 = 0.6$ , with  $p_{\beta,i} = p_{\beta_3,i} = p_{\phi,i} = \sqrt{\frac{1}{3}}$ ,  $\beta_0$  given by (20),  $\beta_{30} = 5$  and  $\phi_0 = 10$ , giving  $\delta = -\sqrt{\frac{1}{3}}$ . The G5 brane appears after a small instanton transition with essentially zero size. It then expands, asymptotically approaching a constant size which is smaller than the length scale associated with the transverse space. This type of evolution occurs in certain, fairly special, examples of our solutions as described in section IV.

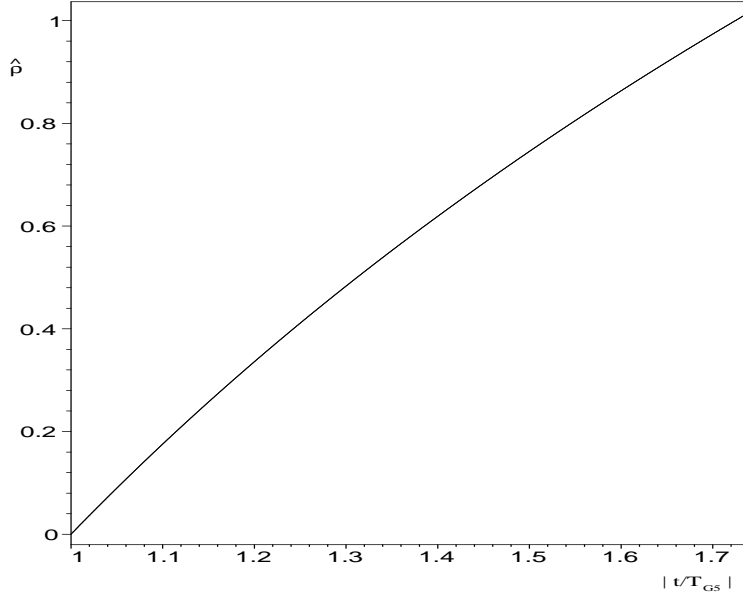


FIG. 2. Plot of the coordinate size of a gauge five brane including the regime where our four dimensional effective theory is valid. We have chosen the set up  $T_{G5} = 0.21$ ,  $\hat{s} = -17.28$  and  $\hat{\rho}_0 = 8.61$ , with  $p_{\beta,i} = 0.43$ ,  $p_{\beta_3,i} = -0.45$ ,  $p_{\phi,i} = 0.87$ ,  $\beta_0 = 6.39$ ,  $\beta_{30} = 17.98$ ,  $\phi_0 = 19.28$  and  $\alpha_0 = -0.51$ . The G5 brane appears after a small instanton transition with essentially zero size. It then expands rapidly, quickly reaching a size comparable to that of the transverse space. At this point our four dimensional theory breaks down and so we have truncated our plot appropriately. This type of evolution is fairly generic for the class of solutions we have presented here as described in section IV.

## B. Solutions including M-five brane moduli

Our goal in presenting new solutions for moving M5 branes in heterotic M-theory in this section is simply to show that the qualitative results found elsewhere [10] also apply to the current case. As such we shall be quite brief in our exposition. The explicit solutions, in comoving gauge, for the case of an M5 brane traversing the bulk are the following.

$$\alpha = \frac{1}{3} \ln \left| \frac{t - t_0}{T} \right| + \alpha_0 \quad (33)$$

$$\beta = p_{\beta,i} \ln \left| \frac{t - t_0}{T} \right| + \beta_0 \quad (34)$$

$$\beta_3 = p_{\beta_3,i} \ln \left| \frac{t - t_0}{T} \right| + (p_{\beta_3,f} - p_{\beta_3,i}) \ln \left( \left| \frac{t - t_0}{T} \right|^{-\delta} + 1 \right)^{-\frac{1}{\delta}} + \beta_{30} \quad (35)$$

$$\phi = p_{\phi,i} \ln \left| \frac{t - t_0}{T} \right| + (p_{\phi,f} - p_{\phi,i}) \ln \left( \left| \frac{t - t_0}{T} \right|^{-\delta} + 1 \right)^{-\frac{1}{\delta}} + \phi_0 \quad (36)$$

$$z = d \left( 1 + \left| \frac{T}{t - t_0} \right|^{\delta} \right)^{-1} + z_0 . \quad (37)$$

The expansion powers in this solution are subject to the following constraint.

$$2p_{\beta,n}^2 + p_{\beta_3,n}^2 + p_{\phi,n}^2 = \frac{4}{3} \quad (38)$$

for  $n = i, f$ . In this expression  $p_{\beta,f} = p_{\beta,i}$ .

In addition to these expansion powers the solution depends upon the arbitrary constants  $T$  and  $t_0$ . There are a further set of constants  $\alpha_0$ ,  $\beta_0$ ,  $\beta_{30}$  and  $\phi_0$  which are also subject to a constraint.

$$\phi_0 - \beta_{30} = \ln \left( \frac{1}{2} d^2 q_{M5} \right) . \quad (39)$$

The constant  $\delta$  is determined by a particular combination of the expansion powers.

$$\delta = p_{\beta_3,i} - p_{\phi,i} . \quad (40)$$

Finally we have the two constants associated with the integration of the five brane's position modulus equation of motion,  $d$  and  $z_0$ .

The solutions given in equations (33) - (37) describe the following physical situation. They again take the form of either positive or negative time branches and, as in the G5 brane case, we take  $\delta > 0$  in the (-) branch and  $\delta < 0$  in the (+) one. The system starts out in a rolling radius solution associated with the theory in the absence of an M5 brane [21]. The M5 brane is initially stationary. Then, at some time determined by  $T$  and  $t_0$ , the M5 brane moves in the orbifold direction until it comes to rest at some new position. During this motion the five brane, through its coupling to the four dimensional metric moduli, changes the initial rolling radius solution into a different one. Thus the system finishes up in some different final rolling radius solution with the five brane again at rest. We can write down a mapping which describes how the moving brane takes one set of expansion powers into another as follows.

$$\begin{pmatrix} p_{\beta,f} \\ p_{\beta_3,f} \\ p_{\phi,f} \end{pmatrix} = P \begin{pmatrix} p_{\beta,i} \\ p_{\beta_3,i} \\ p_{\phi,i} \end{pmatrix} , \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (41)$$

It should be noted that one of the conditions for small warping in the heterotic M-theory vacuum [14] is always violated asymptotically for these solutions (assuming that the five brane moves). In fact it is important to emphasise that in order to keep this warping parameter much less than one in any region of the solution it is necessary that the coordinate distance that the M5 brane moves must be much larger than the size of the orbifold. This arises in a very similar manner to the constraint on  $\hat{s}$  in the previous section. It means that here too we cannot have a single solution which has all of the asymptotic features we have described in a single regime for which the four dimensional theory is valid. An example of the dynamics of a moving five brane as described by our solutions is provided in figure 3 including the portion of the time evolution where the four dimensional theory is valid.

In short we find that these solutions do indeed share all of the qualitative properties of the solutions presented previously for the  $h_{1,1} = 1$  case [10] (although the restriction mentioned above was not made explicit in that paper). These solutions have been described in some detail in the literature and the interested reader is referred there for further discussion.

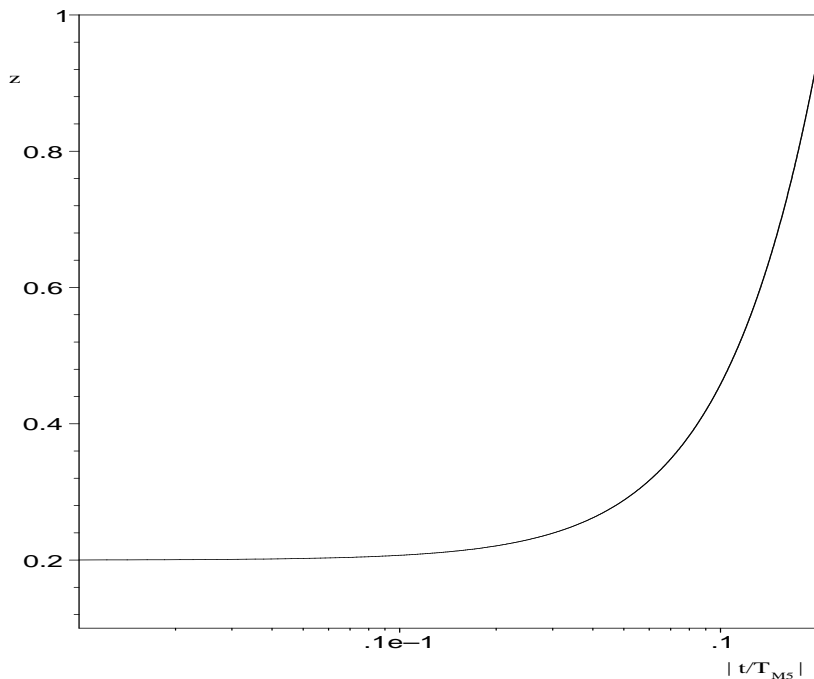


FIG. 3. Plot of the coordinate position of an M5 brane including the regime where our four dimensional effective description is valid. We have chosen the set up  $T_{M5} = 1$ ,  $d = -10$  and  $z_0 = 10.2$ , with  $p_{\beta,i} = 0$ ,  $p_{\beta_3,i} = -\sqrt{\frac{T}{3}}$ ,  $p_{\phi,i} = 1$ ,  $\beta_0 = 5$ ,  $\beta_{30} = 17$ ,  $\alpha_0 = 0$ , and  $\phi_0$  determined by (39). The M5 brane starts out close to the left boundary plane at  $z=0$  and moves across the orbifold direction to collide with the right boundary at  $z=1$ .

#### IV. CONCLUSIONS ABOUT POSSIBLE DYNAMICS AFTER A SMALL INSTANTON TRANSITION AND GENERALISATIONS OF OUR RESULTS

Given the results of the previous section, although we cannot say whether or not the small instanton transition is dynamically allowed, we can make some definite statements about the dynamics of the five branes before and after such a transition.

The dynamics associated with the M5 brane before the collision is the same as would have been expected from analogy with previous work [10]. Consider a case where the M5 brane starts out at some, approximately constant, position in the bulk with the system evolving in some rolling radius solution. Then at some time determined by the integration constants of the system ( $T$  and  $t_0$ ) the M5 brane starts to move as described in equation (37). This causes the other moduli to depart from their 'rolling radius' behaviour as described in equations (35) and (36). The M5 brane moves through the bulk and impacts upon one of the orbifold fixed planes. The fixed plane that is hit can be determined by an appropriate choice of the constants  $d$  and  $z_0$ . This impact marks the end of regime one in our parlance.

There is then a regime which is poorly understood. The tensionless non-critical string comes into play and the dynamical degrees of freedom somehow switch over, we postulate, from those associated with an M5 brane to those associated with the G5 brane. If this G5 brane reaches a width which is larger than the fundamental scale we then enter the third of our regimes.

In this third regime the soliton appears with fundamental length scale width. It then expands monotonically with time in one of two possible ways.

For generically acceptable choices of the integration constants, specifically for most choices which result in the size of the transverse space being much bigger than the fundamental scale, the G5 brane rapidly and monotonically increases in size until it reaches a size comparable to that of the compact manifold it lives in. This can be seen by looking at values of  $\hat{s}$  which are compatible with the constraint as given in (31) and (32) and examining the impact that such choices of integration constant have on the G5 brane evolution via equation (18). Once the G5 brane is comparable in size to the subspace of the Calabi-Yau transverse to it we can no longer trust our four dimensional theory. It is interesting that we are able to say something so specific about the evolution after a small instanton transition, that the gauge field spreads out very rapidly from being a localised lump to being a diffuse configuration. This is also a desirable result from the point of view of the phenomenological models based upon small instanton transitions [22,23]. This is because it means that, in a regime where non-perturbative potentials are unimportant, the G5 brane does not shrink back to zero size again once it has been created and the transition, once it occurs, is permanent. The rapid expansion of the G5 brane induces a complicated evolution in the modulus describing the size of the space transverse to it within the Calabi-Yau, as described in (15).

For certain choices of integration constant however the G5 brane will expand, again from fundamental scale size, to reach a constant width which is smaller than the transverse space. By constant size we mean that the objects coordinate size,  $\hat{\rho}$ , will remain constant, i.e. the ratio of its size to that of the transverse space is fixed. After this the G5 brane will simply scale in size with the transverse space and the four dimensional metric moduli will settle down into some new rolling radius solution. In fact the choice of integration constants required to obtain this behaviour is fairly contrived. If we want the transverse space to the G5 brane in the Calabi-Yau to be at least several orders of magnitude larger than the fundamental scale at its largest then, from equation (31), we must take  $\hat{s} \geq 100v_{\text{trans}}^{\frac{1}{4}}$ . In our conventions however the coordinate size of the transverse space is merely of order  $v_{\text{trans}}^{\frac{1}{4}}$ . Looking at equation (18) we see then that for the final size of the G5 brane,  $\hat{\rho}_0$ , to be smaller than the size of the transverse space we require  $\hat{\rho}_0/\hat{s} \leq 0.01$ .

This possible late time behaviour, with the G5 brane degrees of freedom dropping out of the dynamics, gives some justification to the approach that was taken in several recent papers [22,23] of ignoring the soliton's moduli after the collision. This approach was taken because, at the time, the four dimensional theory including G5 brane moduli was not known. Nevertheless we see that, providing we are looking at the system at times long enough after

the collision, this approach turns out to give a very reasonable approximation to the true physical situation for certain special configurations. In general, however, we should include one of the more generic solutions described above in our analysis of these scenarios. The problem then is that we lose control of the four dimensional effective theory very quickly. This however, is the best that can be done with current technology, and at least it is possible to describe the dynamics on either side of the small instanton transition and show that the G5 rapidly spreads out from its initially small size.

Another general comment we can make is that the size of the transverse space to the G5 brane within the Calabi-Yau always ends up collapsing after a small instanton transition. This follows from equations (15), (21) and (28) and the requirement that the G5 brane grows from its very small initial size so that we can describe it using supergravity. This last means that when our description of regime three begins the G5 brane modulus is increasing and so we cannot pick a solution of constant width. Presumably this collapse is halted by whatever mechanism finally stabilises the extra dimensions (subject to the assumptions we have made about this mechanism here as detailed in the introduction).

To recap, the crucial point is that, due to some generic properties of our evolving G5 brane solutions, we can make some qualitative comments about the dynamics after a small instanton transition despite the fact that, because of our ignorance of regime two, we have no rigorous procedure for matching the solutions in regime one to those in regime three across the transition.

#### A. Consequences of possible sets of matching conditions.

Although we have no microscopic description of the dynamics of regime two we can still make an educated guess as to how to match our solutions for regime one and three across the transition. In many diverse situations where extra light states appear one can match across the transition simply by assuming that the moduli and scale factor are smooth and continuous throughout. One recently studied example is the flop transition [24].

It is easy to see that this procedure is in fact not possible in this case, if we assume that  $z$  and  $\hat{\rho}$  each describe part of one and the same continuous flat direction in moduli space. If we match the values and derivatives of  $\alpha, \beta, \beta_3$  and  $\phi$  at the collision then we find that  $\dot{\hat{\rho}}$  after the collision is determined by this data and the constraint (7). It turns out that, due to the different prefactors of the two five brane moduli in actions (1) and (2), that the value we obtain in this manner for  $\dot{\hat{\rho}}$  after the collision is not the same as the value of  $\dot{z}$  beforehand.

In each of the cases where this naive matching of fields and their derivatives has been shown to be a good approximation the extra light states which have appeared have been particles. In our case the extra light states are associated with a tensionless string. This difference, along with the possibility that  $\hat{\rho}$  and  $z$  do not constitute part of the same flat direction, could be responsible for the fact that the simple matching procedure that has worked in the past does not work in this case. It is interesting that we are able to show so simply and explicitly that the usual procedure fails. In addition, whatever the precise dynamics of the small instanton transition, we conclude that it has to somehow account for the required discontinuity in the fields described above.

The next most simple set of matching conditions would be to match all of the metric moduli, the four dimensional scale factor and these fields' derivatives as before and then to determine the bundle modulus' derivative at collision as described in the previous paragraphs. This, combined with the fact that the G5 brane starts out at 'zero' size is enough to specify a complete set of initial conditions for regime three, given a solution for the moving M5 brane in regime one. It is of interest to see if we can add anything to our general analysis, as presented at the start of this section, if we assume that we can pair up solutions in this particular manner. In fact the data plotted in figures 3 and 2 has been chosen such that the two can be matched together in this way.

Once we have followed this procedure we can describe how the G5 brane evolves after the collision given how the M5 brane was moving before hand. We can use this to elaborate on our expectation, as mentioned above, that the G5 brane will generically evolve as shown in figure 2 as opposed to as shown in figure 1. Generically we can see from (37) that we expect the M5 brane to be moving rapidly at collision. This is essentially because our solution for  $z$  is an 'S' shaped curve implying that the M5 brane, except in two small regimes, is moving rapidly or not at all. Because the M5 brane obviously has to be moving at collision this means that we expect it to be on the steep part of the 'S' when we enter regime two. We may then consider what this means for the resulting initial rate of change of  $\hat{\rho}$  after the collision. The bundle modulus moves in a similar way, also forming an 'S' curve when the complete solution (of which we can physically sample but part) is plotted. By examining the constraints (7)-(12) we can see that if we have a large kinetic energy associated with the M5 brane before collision we would also expect a large kinetic energy for the bundle modulus after collision. In other words we would expect to find our solution in regime three on the steep part of the curve. This adds to and is in accord with comments we have made at the start of this section where we were not considering a particular form of matching.

The plots 3 and 2 for  $z$  and  $\hat{\rho}$  show that, in a particular such example, the M5 brane is indeed moving rapidly at collision and that this results in a rapidly expanding G5 brane in the final regime.

## B. Generalisations of our results.

One possible direction for future research is to include more moduli in our cosmological analysis. We have, in this paper, just included the key moduli for the phenomena we are trying to investigate. There are many other interesting moduli which we could include, for example the moduli which delineate the holomorphic curve which the five branes wrap. We could also generalise our cosmological ansatz to include spatial curvature or, perhaps more interestingly, an ideal gas living on either of the fixed planes. These last two possibilities, for example, could even be solved completely analytically using the Toda theory methods described in [20].

Often in freely moving moduli solutions such as those presented in this paper we can use certain symmetries of the action to generate solutions involving axion fields from solutions which only include the real parts of the various superfields. It is well known [10] that it is no longer possible to include all of the axions in this manner when an M5 brane is present due to the object explicitly breaking some of the relevant symmetry. The same is in fact true in the presence of a gauge five brane and so, due to the numerous cross terms in the relevant action [12], including all of the axions in these solutions would be a difficult challenge.

Finally it would be interesting to try and include the effects on our solutions of the potentials which are present due to gaugino condensation, flux on the internal manifold, and membrane instanton effects [13]. We would expect such solutions to agree with those presented here when the kinetic energy of the moduli fields is sufficiently large (as described in the introduction) but deviate significantly, for example, at late times if we have not reached a large radius limit.

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